MATH 2B/5B Prep: Sigma Notation

1. Write the sum $2 + 5 + 8 + \cdots + 59 + 62$ using sigma notation.

Solution: Notice we go from one number to the next by adding 3. We can establish a pattern

$$\begin{aligned} 2 &= 3(1) - 1 \\ 5 &= 3(2) - 1 \\ 8 &= 3(3) - 1 \\ &\vdots \\ 62 &= 3(21) - 1 \end{aligned}$$

So we define $a_i = 3i - 1$, starting at $i = 1$ and ending at $i = 21$. Therefore

$$2 + 5 + 8 + \cdots + 59 + 62 = \sum_{i=1}^{21} 3i - 1$$

2. Write $\sum_{i=1}^{8} i^2 + 1$ as a sum.

Solution: First we will write out all the terms by plugging the numbers 1 through 8 into the formula $i^2 + 1$:

i	i^2+1	i	i^2+1
1	2	5	26
2	5	6	37
3	10	7	50
4	17	8	65

Then we add up all the terms and get

$$\sum_{i=1}^{8} i^2 + 1 = 2 + 5 + 10 + 17 + 26 + 37 + 50 + 65$$

3. Write out and simplify $\sum_{i=1}^{n} \frac{1}{i} - \frac{1}{i+1}$ for $n = 1, 2, 3, 4$. What is the pattern? What happens as n goes to infinity?

Solution:

$$n = 1: \quad \sum_{i=1}^{1} \frac{1}{i} - \frac{1}{i+1} = 1 - \frac{1}{2}$$

$$n = 2: \quad \sum_{i=1}^{2} \frac{1}{i} - \frac{1}{i+1} = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) = 1 - \frac{1}{3}$$

$$n = 3: \quad \sum_{i=1}^{3} \frac{1}{i} - \frac{1}{i+1} = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) = 1 - \frac{1}{4}$$

$$n = 4: \quad \sum_{i=1}^{4} \frac{1}{i} - \frac{1}{i+1} = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) = 1 - \frac{1}{5}$$

Notice the pattern here is

$$\sum_{i=1}^{n} \frac{1}{i} - \frac{1}{i+1} = 1 - \frac{1}{n+1}$$

As n goes to infinity the second term goes to zero and so

$$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{i} - \frac{1}{i+1} = 1$$